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Near-extremal correlators and generalized consistent truncation for $AdS_{4|7} \times S^{7|4}$

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Abstract: We present conjectures for the space-time form and leading large-N dependence of extremal and near-extremal correlation functions in the $\mathcal{N} = 8$ superconformal Yang-Mills theory in $d = 3$ as well as in the $(0, 2)$ superconformal theory in $d = 6$, using their gravity duals with M-theory on $AdS_4 \times S^7$ and $AdS_7 \times S^4$ respectively. As a key part of the conjectures, we argue that the bulk couplings associated with extremal and near-extremal field configurations in the corresponding AdS_4 and AdS_7 gauged supergravities vanish. The vanishing of these couplings constitutes a generalization of the property of consistent truncation of the Kaluza-Klein modes.

Keywords: AdS-CFT Correspondence, Conformal Models in String Theory, Conformal and W Symmetry.

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1. Introduction

$\mathcal{N} = 4$ SYM and type-IIB supergravity on $AdS_5 \times S^5$. A number of remarkable conjectures on the factorization and coupling constant dependence of correlators of local chiral operators have emerged from the AdS/CFT conjecture [1, 2] between type-IIB superstring theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ superconformal Yang-Mills theory on $\mathbb{R}^4$. One important discovery is that certain correlators exhibit a special factorized space-time form and are independent of the Yang-Mills coupling, g_{YM}, a phenomenon usually referred to as "non-renormalization". Another surprising result is that certain associated supergravity couplings in $AdS_5 \times S^5$ vanish, thereby extending the usual property of consistent truncation. Most results obtained so far are on the correlators of 1/2 BPS operators of $\mathcal{N} = 4$ superconformal Yang-Mills theory, i.e. the theory that corresponds to type-IIB superstring theory on $AdS_5 \times S^5$. The superconformal primary operators of this theory are denoted by $\mathcal{O}_\Delta$ with dimension Δ and $SU(4)$ Dynkin label $(0, \Delta, 0)$. Normalizing the operators $\mathcal{O}_\Delta$ by their 2-point functions as usual, the following conjectures have been proposed:

Conjecture I: $\mathcal{N}=4$ SYM correlators.

1. Non-renormalization of three-point functions $\langle\mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\mathcal{O}_{\Delta_3}(x_3)\rangle$ of single and multiple color trace 1/2 BPS operators $\mathcal{O}_{\Delta_i}$;

2. Non-renormalization and factorization into a product of n two-point functions of *extremal correlators* $\langle\mathcal{O}_{\Delta}(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle$ of single and multiple trace 1/2 BPS operators $\mathcal{O}_{\Delta_i}$, whose dimensions satisfy $\Delta=\Delta_1+\cdots+\Delta_n$;

3. Non-renormalization and decomposition into a sum of products of a single non-renormalized three-point function and $n-1$ non-renormalized two-point functions of *next-to-extremal correlators* $\langle\mathcal{O}_{\Delta}(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle$ of single trace 1/2 BPS operators $\mathcal{O}_{\Delta_i}$, whose dimensions satisfy $\Delta+2=\Delta_1+\cdots+\Delta_n$;

4. Decomposition into a sum of products of non-renormalized two- and three-point functions and (in general, renormalized) higher point functions of *sub-extremal correlators* $\langle\mathcal{O}_{\Delta}(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle$ of single 1/2 BPS operators $\mathcal{O}_{\Delta_i}$ whose dimensions satisfy $\Delta+2m=\Delta_1+\cdots+\Delta_n$ with $2\leq m\leq n-2$.

The evidence for (1) derives from a detailed comparison between the 2- and 3-point functions calculated on the AdS supergravity side, (using the values for the supergravity couplings derived from the supergravity field equations and action) and the free field values for the same correlators on the superconformal Yang-Mills side [3, 4]. The AdS supergravity expansion is valid for large-N and large 't Hooft coupling $\lambda=g_{YM}^2N$, so that equality between the correlators in the two regimes supports the conjecture (1) at least in the large-N limit. Evidence for (1) for all values of N is secured from the fact that 2- and 3-point functions suffer no corrections to first order in g_{YM}^2 [5, 6]. Meanwhile, it was shown that 2-points also suffer no corrections to order $\mathcal{O}(g_{YM}^4)$ [7]. Finally, methods of $\mathcal{N}=2$ analytic superspace [8], combined with a recursive equation of [9], and $\mathcal{N}=4$ superconformal invariance properties of the correlators provide arguments that come close to being a general proof of conjecture (1). (A loose end in the argument is related with the existence of certain contact invariants, whose contribution has not been ruled out so far.[1])

Part (2) of the conjecture was first proposed in [10] on the basis of evidence gathered from the AdS supergravity side. A priori, this evidence appears weaker than it was in the case of part (1), since the explicit form of the AdS gauged supergravity couplings has not been evaluated directly from the supergravity action. Remarkably however, there is an indirect argument that the supergravity couplings for extremal arrangements of $n+1$-point couplings vanish as a consequence of the finiteness of superstring theory on $AdS_5\times S^5$ at the *string tree level*, as well as the fact that a Maldacena dual field theory exists at the boundary of AdS_5. Indeed, the

[1]We are grateful to Marc Grisaru for a discussion on this point. See also [7].

AdS contact integrals appearing in the evaluation of the correlators diverges precisely when the dimensions of the fields are extremal, so that the associated supergravity couplings must vanish [10]. (See also [11] where this possibility was mentioned.) The extremal 4-point function has been shown to vanish by explicit calculation in [12], thus confirming the proposed pattern. Meanwhile, further evidence for (2) has accumulated from perturbative calculations [14] and from $\mathcal{N}=2$ analytic superspace arguments [13].

Part (3) was proposed in [13] based on $\mathcal{N}=2$ analytic superspace arguments, while further evidence from perturbation theory and AdS supergravity were given in [15]. The 4-point next-to-extremal supergravity couplings were shown to vanish in [16]. Part (4) was proposed in [17] based on perturbative and AdS supergravity arguments, which furthermore lead to a set of challenging conjectures on the structure of the AdS gauged supergravity theory:

Conjecture II: $AdS_5 \times S^5$ **vanishing near-extremal couplings.**

- The bulk supergravity couplings $\mathcal{G}(\Delta, \Delta_1, \ldots, \Delta_n)$ between the fields s_{Δ_i} dual to the 1/2 BPS single trace operators $\mathcal{O}_{\Delta_i}$ vanish for *near-extremal* arrangements of dimensions $\Delta + 2m = \Delta_1 + \cdots + \Delta_n$ whenever $0 \leq m \leq n-2$,

$$\mathcal{G}(\Delta, \Delta_1, \ldots, \Delta_n) = 0\,; \tag{1.1}$$

Note that this cancellation cannot merely be a property of $\mathrm{SU}(2,2|4)$ superconformal invariance, since the superconformal Yang-Mills correlators $\langle \mathcal{O}_\Delta(x)\mathcal{O}_{\Delta_1}(x_1)\ldots \mathcal{O}_{\Delta_n}(x_n)\rangle$ in identically the same $\mathrm{SU}(2,2|4)$ representation are non-vanishing.

There is further evidence for the validity of the $AdS_5 \times S^5$ supergravity conjecture II from the well-known property of *consistent truncation* of AdS gauged supergravity. Consistent truncation means that Kaluza-Klein excitation modes of type IIB on $AdS_5 \times S^5$ can systematically decouple from the pure AdS_5 gauged supergravity, since the latter is a consistent theory by itself [18]. This implies that the supergravity couplings between n pure AdS supergravity fields and any single Kaluza-Klein excitation field must vanish, i.e. for all $n \geq 1$ we have

$$\mathcal{G}(\Delta, 2_1, \ldots, 2_n) = 0\,. \tag{1.2}$$

For $\Delta > 2n$, $\mathrm{SU}(4)_R$ group theory automatically guarantees the vanishing of these couplings anyway, while, $\mathrm{SU}(4)_R$ quadrality requires Δ even. Thus, the non-trivial conditions imposed by consistent truncation are the vanishing of the supergravity couplings for $\Delta = 4, 6, \ldots, 2n$, which is precisely the contents of the conjecture (1.1) for the dimensions $\Delta_i = 2$.

Though consistent truncation is by now a well-established property of the supergravity field equations, a deeper understanding of its geometrical significance still

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appears desirable. The $AdS_5 \times S^5$ supergravity conjecture II (1.1) provides a challenging generalization of standard consistent truncation. As the vanishing of supergravity couplings in (1.1) is not merely a consequence of $\mathrm{SU}(2,2|4)$ group theory, its existence may point the way to additional hidden symmetries of gauged supergravity.

$d = 3, 6$ **SCFT and 11-dimensional SUGRA on** $AdS_{4|7} \times S^{7|4}$**.** In the present paper, we investigate to what extent factorization, non-renormalization and generalized consistent truncation conjectures hold for the AdS/CFT correspondences for M-theory on $AdS_4 \times S^7$ and $AdS_7 \times S^4$. The 11-dimensional supergravity on these spaces was considered long ago in [20, 21, 22, 23] and [22, 24, 25] respectively. The property of consistent truncation was investigated in [25, 26].

The associated superconformal field theory duals, $\mathcal{N} = 8$ superconformal Yang-Mills in $d = 3$ and $(0, 2)$ superconformal "gauge theory" in $d = 6$ respectively, are considerably less understood than their $\mathcal{N} = 4$, $d = 4$ counterpart [27]. The key difficulty is the absence of a freely adjustable coupling constant or marginal deformation. Instead, both theories emerge as isolated strong coupling fixed points, with only the size N of the gauge group $\mathrm{SU}(N)$ as a free parameter. Moreover, finite temperature computations, absorption cross-sections and anomalies reveal that the effective number of degrees of freedom in the large-N limit behaves as $N^{3/2}$ for $d = 3$ and as N^3 for $d = 6$, both radically different from the customary field theory results [28]. On the AdS side, the absence of marginal deformations implies that the $1/N$ expansion will govern both the quantum corrections and the low energy expansion of M-theory on $AdS_4 \times S^7$ or on $AdS_7 \times S^4$.

The absence of any marginal deformations to the $d = 3$ and $d = 6$ theories renders the issue of "non-renormalization" of AdS/CFT correlators moot, as there simply is no free coupling constant to be considered. However, keeping in mind the analogy with the $AdS_5 \times S^5$ case, the questions of "factorization" as well as of the "vanishing of near-extremal supergravity couplings" continue to be challenging. It is these questions that we shall investigate here. In particular, we shall present evidence for the following conjecture.

Conjecture III: $AdS_{4|7} \times S^{7|4}$ **vanishing near-extremal couplings.**

- The bulk supergravity couplings $\mathcal{G}(\Delta, \Delta_1, \ldots, \Delta_n)$ between the fields s_{Δ_i} dual to the 1/2 BPS single trace operators $\mathcal{O}_{\Delta_i}$ vanish for *near-extremal* arrangements of dimensions $\Delta + 2m\mathcal{K} = \Delta_1 + \cdots + \Delta_n$ whenever $0 \le m \le n-2$,

$$\mathcal{G}(\Delta, \Delta_1, \ldots, \Delta_n) = 0\,; \tag{1.3}$$

Here $\mathcal{K}$ is the unit of dimension for superconformal primaries, $\mathcal{K} = 1/2$ for AdS_4 and $\mathcal{K} = 2$ for AdS_7. Just as in the case of type-IIB supergravity on $AdS_5 \times S^5$, this conjecture encompasses the vanishing relations that are equivalent to consistent truncation, and generalizes consistent truncation in a non-trivial way.

Our main argument for this conjecture will come from the finiteness of the extremal correlator of boundary superconformal primaries: on the AdS gravity side, this correlator arises from a number of tree-level exchange diagrams plus one contact diagram. All exchange diagrams are finite, but the integral on the position of the vertex in the contact diagram diverges when the vertex approaches the boundary insertion with highest conformal dimension. Hence the vertex should vanish for the correlator to be finite. While this argument will be spelled out in detail for the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ cases in section 4, it should be noted that it is expected to apply more generally for any weakly coupled supergravity on a product space $AdS_{d+1} \times \mathcal{M}$, provided this theory admits a finite conformal quantum field theory dual. Assuming that a discrete spectrum of operators $\mathcal{O}_\Delta$ exists, (as expected for $\mathcal{M}$ compact) the supergravity coupling between the dual fields s_{Δ_i} whose exact quantum dimensions satisfy $\Delta = \Delta_1 + \cdots + \Delta_n$ will be forced to vanish by the same reasoning as above. Such linear relations between dimensions are the rule in supersymmetric theories, but it is conceivable that the argument could be generalized to situations with softly or spontaneously broken supersymmetry as well. It is expected to apply in particular to type-IIB superstrings on $AdS_3 \times S^3 \times \mathbb{T}^4$ with 32 supercharges, to $AdS_3 \times S^3 \times K_3$ with 16 supercharges and to $AdS_5 \times S^5/\Gamma$ with 16 or 8 supercharges amongst other examples.

Outline. The remainder of the paper is organized as follows. In section 2, we review the $d = 3, 6$ superconformal theories, including the structure of their 1/2 BPS operators. In section 3, we present general convergence and divergence criteria for tree level integrals on AdS space-times. In section 4, we use the assumption of finiteness to show that extremal supergravity couplings must vanish, and we derive a general factorized form of extremal correlation functions 1/2 BPS operators. In section 5, we present conjectures on the factorization and vanishing of near-extremal supergravity couplings.

2. The $d = 3, 6$ superconformal theories

The $d = 3$ and $d = 6$ superconformal theories with 16 supercharges describe the world-volume dynamics of a stack of N coincident M2-branes or M5-branes embedded in flat eleven-dimensional Minkowski space. These interacting theories can be obtained by a renormalization flow from the more familiar supersymmetric gauge theories describing the dynamics of non-coincident D2 and D4 branes at weak coupling respectively (see [29] for a review). Indeed, the three-dimensional $\mathcal{N} = 8$ super-Yang-Mills theory living on the coincident D2-branes is strongly coupled at energies much smaller than $g^2_{YM3} = g_s/l_s = l_p^3/R_s^2$, where l_p is the eleven-dimensional Planck length and $R_s = g_s l_s$ the size of the eleven dimension, and flows to an interacting infrared fixed point with $\mathcal{N} = 8$ superconformal symmetry in the eleven-dimensional decompactification limit $R_s/l_p \to 0$ [30]. On the contrary, the five-dimensional $\mathcal{N} = 4$ super-Yang-Mills theory on the coincident D4-branes is gaussian in the infrared, but strongly coupled at energies much bigger than $1/g^2_{YM5} = 1/(g_s l_s) = 1/R_s$. At this scale, a new dynamically generated dimension opens up, whose momentum excitations are the Yang-Mills instantons of mass $1/g^2_{YM5}$ [31]. The ultraviolet behaviour is controlled by a $d = 6$ $(0, 2)$ superconformal theory, which is also the world-volume theory of the M5-brane. The same theory can also be obtained as the decoupling limit $g_s \to 0, l_s \to 0$ of the type-IIA NS5-brane [32], or as the type-IIB string theory compactified on a A_{N-1} singularity [33]. We now briefly recall properties of these superconformal fixed points, based on their symmetry algebras, which are two different real forms of the orthosymplectic superalgebra OSp(8|4), whose representations can be analyzed as in [34].

2.1 The $d = 3$ $N = 8$ superconformal theory

The $d = 3$ theory has conformal group $\mathrm{SO}(3, 2) \sim \mathrm{Sp}(4)$, extended with 8 odd generators in the pseudo-real four-dimensional spinor representation of $\mathrm{SO}(3, 2)$, rotated into each other[2] by an R-symmetry SO(8). In the $d = 3$ SYM theory, only a SO(7) subgroup of SO(8) is realized linearly, and the full SO(8) symmetry emerges dynamically only in the strong coupling limit. The theory contains one gauge field A_μ, one fermion λ^a in the spinor representation **8** of SO(7) and seven scalars X^I, $I = 1, \ldots, 7$, in the vector representation **7**, all taking values in the Lie algebra of SU(N). The microscopic lagrangian is given by

$$\mathcal{L} = \frac{1}{g^2_{YM3}} \mathrm{tr} \left\{ F^2_{\mu\nu} + (\nabla_\mu X^I)^2 + [X^I, X^J]^2 + \lambda^a \gamma^\mu \nabla_\mu \lambda^a + \lambda^a \Gamma^I_{ab} X^I \lambda^b \right\}, \tag{2.1}$$

where Γ^I and γ_μ are SO(7) (internal) and SO(2,1) (space-time) gamma matrices respectively. In the free theory, the gauge field can be dualized into an eighth scalar X^8, in terms of which the free lagrangian exhibits the full SO(8) R-symmetry. The field content is now 8 scalars X^i, $i = 1, \ldots, 8$, and eight fermions λ_a, $a = 1, \ldots, 8$. Denoting the SO(8) gamma matrices by Γ^i, we have the following supersymmetry transformations,

$$\delta X^i = -i\epsilon^a \Gamma^i_{ab} \lambda^b \,, \qquad \delta\lambda_a = \Gamma^i_{ab} \gamma^\mu \partial_\mu X^i \epsilon^b \,. \tag{2.2}$$

The complete spectrum at the infrared fixed point is not known, but the chiral (or BPS) operators can be followed from weak coupling, since their dimensions are protected from quantum corrections. They form infinite dimensional unitary representations of OSp(2, 6|4), for which oscillator [24] or harmonic superspace constructions [35] are available. These representations are built by applying the supersymmetry generators on a lowest-weight vector (or superconformal primary operator,

[2]We take the convention that the supersymmetry charges transform as a spinor $\mathbf{8_s}$ of SO(8).

Free Field Operator	desc	SUGRA$_4$	2× dim	spin^P	SO(8)	lowest reps
$\mathcal{O}_k \sim X^k$, $k \geq 2$	–	h^α_α, $D_\alpha D_\beta h^{\alpha\beta}$	k	0^+	$(k,0,0,0)$	$\mathbf{35}_v$,$\mathbf{112}_v$
$\mathcal{O}^{(1)}_k \sim \lambda X^k$, $k \geq 1$	Q	ψ_α	$k+2$	$1/2$	$(k,0,1,0)$	$\mathbf{56}_s$,$\mathbf{224}_{vc}$
$\mathcal{O}^{(2)}_k \sim \lambda\lambda X^k$	Q^2	$a_{\alpha\beta\gamma}$	$k+4$	0^-	$(k,0,2,0)$	$\mathbf{35}_c$,$\mathbf{224}_{cv}$
$\mathcal{O}^{(3)}_k \sim \lambda\lambda X^k$	Q^2	$h_{\mu\alpha}$, $a_{\mu\alpha\beta}$	$k+4$	1^-	$(k,1,0,0)$	$\mathbf{28}$,$\mathbf{160}_v$
$\mathcal{O}^{(4)}_k \sim \lambda\lambda\lambda X^k$	Q^3	ψ_α	$k+6$	$1/2$	$(k,1,1,0)$	$\mathbf{160}_c$,$\mathbf{840}_s$
$\mathcal{O}^{(5)}_k \sim \lambda\lambda\lambda X^k$	Q^3	ψ_μ	$k+6$	$3/2$	$(k,0,0,1)$	$\mathbf{8}_s$,$\mathbf{56}_c$
$\mathcal{O}^{(6)}_k \sim (\partial X)^2 X^k$	Q^4	$h_{\mu\nu}$	$k+6$	2^+	$(k,0,0,0)$	$\mathbf{1}$,$\mathbf{8}_v$, $\mathbf{35}_v$
$\mathcal{O}^{(7)}_k \sim (\partial X)\lambda\lambda X^k$	Q^4	$a_{\mu\alpha\beta}$	$k+7$	1^+	$(k,0,1,1)$	$\mathbf{56}_v$,$\mathbf{350}$
$\mathcal{O}^{(8)}_k \sim \lambda\lambda\lambda\lambda X^k$	Q^4	$h_{(\alpha\beta)}$	$k+8$	0^+	$(k,2,0,0)$	$\mathbf{300}$,$\mathbf{1400}$
$\mathcal{O}^{(9)}_k \sim (\partial X)^2 \lambda X^k$	Q^5	ψ_μ	$k+8$	$3/2$	$(k,0,1,0)$	$\mathbf{8}_c$,$\mathbf{56}_s$
$\mathcal{O}^{(10)}_k \sim (\partial X)\lambda\lambda\lambda X^k$	Q^5	ψ_α	$k+9$	$1/2$	$(k,1,0,1)$	$\mathbf{160}_s$,$\mathbf{840}_c$
$\mathcal{O}^{(11)}_k \sim (\partial X)^2\lambda\lambda X^k$	Q^6	$h_{\mu\alpha}$, $a_{\mu\alpha\beta}$	$k+10$	1^-	$(k,1,0,0)$	$\mathbf{28}$,$\mathbf{160}_v$
$\mathcal{O}^{(12)}_k \sim (\partial X)^2\lambda\lambda X^k$	Q^6	$a_{\alpha\beta\gamma}$	$k+10$	0^-	$(k,0,0,2)$	$\mathbf{35}_c$,$\mathbf{224}_{cv}$
$\mathcal{O}^{(13)}_k \sim (\partial X)^3\lambda X^k$	Q^7	ψ_α	$k+11$	$1/2$	$(k,0,0,1)$	$\mathbf{8}_s$,$\mathbf{56}_c$
$\mathcal{O}^{(14)}_k \sim (\partial X)^4 X^k$	Q^8	h^α_α, $D_\alpha D_\beta h^{\alpha\beta}$	$k+12$	0^+	$(k,0,0,0)$	$\mathbf{1}$,$\mathbf{8}_v$, $\mathbf{35}_v$

Table 1: $AdS_4 \times S^7$ Supergravity fields and $SO(3,2) \times SO(8)$ quantum numbers. The range of k is $k \geq 0$, unless otherwise specified.

SCPO). This yields a finite number of chiral primary operators (CPO), each of which heads an infinite tower of conformal descendents. Here, we shall be interested in 1/2 BPS operators only, defined by

$$\mathcal{O}_k = \mathrm{tr} X^k \equiv \mathrm{Str}(X^{i_1} \cdots X^{i_k}) \,, \tag{2.3}$$

with dimension $\Delta = k/2$ protected from quantum corrections. Str stands for the symmetrized color trace, and it is assumed that the indices i_j are made traceless. The fields X^i denotes the free field with SO(8) Dynkin label $(k,0,0,0)$.[3] Its superconformal descendents are listed in table 1, and their dimensions are easily computed from the free field dimensions $[X] = 1/2$, $[\lambda] = 1$, $[F] = 3/2$. Non-symmetric or traceful representations do not give rise to 1/2 BPS operators. On the other hand, multi-trace operators in the same $(k,0,0,0)$ are also 1/2 BPS operators [36], and mix with the single trace ones only at subleading order in $1/N$.

2.2 The $d = 6$ $(0,2)$ superconformal theory

The situation with the $d = 6$ $(0,2)$ superconformal theory is very similar. The conformal group $SO(6,2)$ is extended with 4 odd generators transforming as symplectic-Weyl-Majorana spinors of $SO(6,2)$, rotated into each other by an R-symmetry $USp(4) \sim SO(5)$. In contrast to the $d = 3$ case, the D4-brane theory exhibits the full

[3]Hence $\mathcal{O}_k$ carries a charge $(-)^k$ under the subgroup $\mathbb{Z}_2$ in the center $\mathbb{Z}_2 \times \mathbb{Z}_2$ of Spin(8).

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R-symmetry, since it contains one gauge field A_μ, five scalars X^I in the **5** of USp(4) and four pseudoreal fermions λ in the **4** of USp(4). It is however more convenient to describe the spectrum in terms of the free $d = 6$ $(0, 2)$ tensor multiplet, which makes the six-dimensional Lorentz symmetry manifest. This theory contains one two-form with self-dual field strength $H^+_{\mu\nu\rho}$, which reduces to the $d = 5$ field strength $F_{\mu\nu} = H_{\mu\nu 5}$, four symplectic Majorana-Weyl fermions in the **4** of USp(4), and five scalars. The supersymmetry transformations in the free theory are given by [37]

$$\begin{aligned} \delta X^I &= -\epsilon^a \Gamma^I_{ab} \lambda^b \ , \quad \delta H^+_{\mu\nu\rho} = \epsilon_a \Gamma^{ab}_{[\mu\nu} \partial_{\rho]} \lambda_b \, , \\ \delta \lambda_a &= \frac{1}{4} \gamma^\mu \partial_\mu X^I \Gamma^I_{ab} \epsilon^b - \frac{1}{12} H^+_{\mu\nu\rho} \Gamma^{\mu\nu\rho}_{ab} \epsilon^b \end{aligned} \tag{2.4}$$

All fields take values in the Lie algebra of SU(N), although it is as yet unknown how to consistently switch on interactions. Still it is possible to follow the spectrum of BPS operators from zero-coupling to the superconformal ultraviolet fixed point. The 1/2 BPS chiral operators are obtained from the superconformal primary operator

$$\mathcal{O}_k = \mathrm{tr} X^k \equiv \mathrm{Str}(X^{i_1} \cdots X^{i_k}) \tag{2.5}$$

now in the symplectic-traceless completely symmetric tensor product of k fundamentals of USp(4), with Dynkin labels $(0, k)$ and dimension $2k$. In particular, $\mathcal{O}_k$ carries a charge $(-)^k$ under the center of USp(4). The dimensions of the descendents are computed using the free-field dimensions $[X] = 2, [\lambda] = 5/2$ and $[H_+] = 3$.

2.3 AdS dual and consistent truncation

The BPS spectrum is the only information about the superconformal fixed point that can be reliably computed using the free field theories. On the other hand, Maldacena's conjecture gives a dual description of the large-N limit of these $d = 3$ and $d = 6$ fixed points, in terms of eleven-dimensional supergravity in the near-horizon geometry of the M2 and M5-brane, namely $AdS_4 \times S^7$ and $AdS_7 \times S^4$ respectively [1]. This description is reliable when the radius of the 4-sphere $R_4 = N^{1/6} l_p$ or 7-sphere $R_7 = N^{1/3} l_p$ is much larger than the Planck length l_p, i.e. at large-N. The spectrum of chiral primary fields on the gauge theory is easily matched [27] to the spectrum of Kaluza-Klein states of 11d supergravity compactified on the sphere, which was worked out in [20] and [22] respectively, as shown in tables 1 and 2. The operator on the gauge theory side is represented by its free-field version, using the dualized representation for $d = 3$ and the self-dual tensor multiplet for $d = 6$, and a symmetrized traceless trace in the adjoint representation is understood. In both cases, the operator with $k = 1$ is a pure gauge (doubleton) mode on the supergravity side, which justifies looking at SU(N) theories only. The operator with $k = 2$ and its descendents correspond to the $\mathcal{N} = 8$ supergravity multiplet in the limit of flat Minkowski space, and include the graviton, gravitini, gauge fields, fermions and scalars in the

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Free field Operator	desc	SUGRA$_7$	dim	SU(4)*	USp(4)	lowest reps
$\mathcal{O}_k \sim X^k,\ k \geq 2$	–	${h_\alpha}^\alpha$	$2k$	$(0,0,0)$	$(0,k)$	**5**,**14**,**30**
$\mathcal{O}_k^{(1)} \sim \lambda X^k,\ k \geq 1$	Q	ψ_α	$2k+\frac{5}{2}$	$(1,0,0)$	$(1,k)$	**4**,**16**,**40**
$\mathcal{O}_k^{(2)} \sim H_+ X^k$	Q^2	$A_{\mu\nu\rho}$	$2k+3$	$(2,0,0)$	$(0,k)$	**1**,**5**,**14**
$\mathcal{O}_k^{(3)} \sim \lambda\lambda X^k$	Q^2	$h_{\mu\alpha}\ B_{\mu\alpha}$	$2k+5$	$(0,1,0)$	$(2,k)$	**10**,**35**,**81**
$\mathcal{O}_k^{(4)} \sim H_+\lambda X^k$	Q^3	ψ_α	$2k+\frac{11}{2}$	$(1,0,0)$	$(1,k)$	**4**,**16**,**40**
$\mathcal{O}_k^{(5)} \sim \lambda\lambda\lambda X^k$	Q^3	ψ_μ	$2k+\frac{15}{2}$	$(1,1,0)$	$(3,k)$	**20**,**64**, **140**
$\mathcal{O}_k^{(6)} \sim H_+^2 X^k$	Q^4	$h_{\mu\nu}$	$2k+6$	$(0,2,0)$	$(0,k)$	**1**,**5**,**14**
$\mathcal{O}_k^{(7)} \sim H_+\lambda\lambda X^k$	Q^4	$A_{\mu\alpha\beta}\ B_{\mu\alpha}$	$2k+8$	$(1,0,1)$	$(2,k)$	**10**,**35**,**81**
$\mathcal{O}_k^{(8)} \sim \lambda\lambda\lambda\lambda X^k$	Q^4	$h_{(\alpha\beta)}$	$2k+10$	$(0,0,0)$	$(4,k)$	**35'**,**105**
$\mathcal{O}_k^{(9)} \sim H_+^2\lambda X^k$	Q^5	ψ_μ	$2k+\frac{17}{2}$	$(1,1,0)$	$(1,k)$	**4**,**16**,**40**
$\mathcal{O}_k^{(10)} \sim H_+\lambda\lambda\lambda X^k$	Q^5	ψ_α	$2k+\frac{21}{2}$	$(1,0,0)$	$(3,k)$	**20**,**64**, **140**
$\mathcal{O}_k^{(11)} \sim H_+^3 X^k$	Q^6	$h_{\mu\alpha}\ B_{\mu\alpha}$	$2k+9$	$(0,1,0)$	$(0,k)$	**1**,**5**,**14**
$\mathcal{O}_k^{(12)} \sim H_+^2\lambda\lambda X^k$	Q^6	$A_{\mu\nu\rho}$	$2k+11$	$(1,0,1)$	$(2,k)$	**10**,**35**,**81**
$\mathcal{O}_k^{(13)} \sim H_+^3\lambda X^k$	Q^7	ψ_α	$2k+\frac{23}{2}$	$(1,0,0)$	$(1,k)$	**4**,**16**,**64**
$\mathcal{O}_k^{(14)} \sim H_+^4 X^k$	Q^8	${h_\alpha}^\alpha$	$2k+12$	$(0,0,0)$	$(0,k)$	**1**,**5**,**14**

Table 2: $AdS_7 \times \mathrm{S}^4$ Supergravity fields and $\mathrm{SO}(6,2) \times \mathrm{USp}(4)$ quantum numbers. The range of k is $k \geq 0$, unless otherwise specified.

$\mathbf{1} \oplus \mathbf{8_s} \oplus \mathbf{28} \oplus \mathbf{56_s} \oplus \mathbf{35_v} \oplus \mathbf{35_c}$ for the M2-brane, and $\mathbf{1} \oplus \mathbf{4} \oplus \mathbf{10} \oplus \mathbf{16} \oplus \mathbf{14}$ for the M5-brane. Multitrace operators correspond to multiparticle states on the gravity side.

In addition to yielding the spectrum at the superconformal point, the AdS dual description also allows to extract correlation functions between chiral primaries: in the large-N limit, they are given by diagrams whereby the boundary fields propagates from the boundary to the bulk and interact locally as well as by exchange of bulk modes. This computation requires identifying the gravity modes to which the gauge theory operators couple, and their n-point couplings in the bulk supergravity. Following the $AdS_5 \times S^5$ computation of [3], three-point functions for superconformal primaries have been extracted in [38]. This has been extended to a class of superconformal descendents in [39]. In particular, these results show that extremal 3-point functions vanish in the large-N approximation. Three-point correlators of stress tensors have also been computed using the AdS/CFT correspondence, and turn out to be given by the free-field result [40].

The vanishing of extremal 3-point functions implies in particular that the coupling between two massless states and a massive Kaluza-Klein state vanishes. This fact raises the possibility of truncating the spectrum in order to define a reduced gauge supergravity theory on AdS_4 or AdS_7, as is customary in Kaluza-Klein reductions on tori. Indeed, it has been shown by de Wit and Nicolai long ago in the

$AdS_4 \times S^7$ case [25], and more recently in $AdS_7 \times S^4$ [26], that one may truncate the Kaluza-Klein spectrum to the massless modes in such a way that any solution of the truncated theory can be lifted to a solution of 11d supergravity. The vanishing of extremal 3-point couplings is only a necessary condition for this to hold, and more generally all couplings involving one massive mode and n massless modes should vanish, as in (1.2). In this paper, we shall argue for an even more general decoupling occurring for near-extremal configurations, which hints to a deeper formulation of consistent truncation.

3. Convergence Criteria for AdS integrals

As for the treatment of near-extremal correlation functions in the $AdS_5 \times S^5$ case [10, 17], we shall also here make heavy use of the divergence properties of AdS_{d+1} integrals, in particular for $d = 3, 6$. Thus, we extend the arguments of [10, 17] to general d in order to emphasize that the divergence structure is independent of d.

Throughout, we analytically continue to euclidean AdS which may be represented by the upper half space $AdS_{d+1} = \{(z_0, \vec{z}).\ z_0 > 0,\ \vec{z} \in \mathbb{R}^d\}$, with the Poincaré metric $ds^2 = (dz_0^2 + d\vec{z}^2)/z_0^2$. All AdS integrals of interest to us are associated with correlation functions of the superconformal primary operator of a 1/2 BPS multiplet, which is always a space-time scalar. Thus, the only boundary-bulk propagators that we need are scalar and for a dimension Δ operator are given by

$$K_\Delta(\vec{x}, z) = C_\Delta \Big(\frac{z_0}{z_0^2 + (\vec{z} - \vec{x})^2}\Big)^\Delta \qquad C_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2}\Gamma(\Delta - d/2)}\,, \tag{3.1}$$

for $\Delta > d/2$ and $C_{d/2} = \Gamma(d/2)/2\pi^{d/2}$. Divergences in AdS integrals can arise only when one or several integration points approach the boundary. As a bulk point z approaches the boundary, we have the following behaviors

$$\begin{aligned} K_\Delta(\vec{x}, z) &\to C_\Delta z_0^\Delta \frac{1}{(\vec{z} - \vec{x})^{2\Delta}} \qquad z_0 \to 0\,,\ \vec{z} \neq \vec{x}\,, \\ K_\Delta(\vec{x}, z) &\to z_0^{4-\Delta} \delta(\vec{z} - \vec{x}) \qquad \vec{z} \to \vec{x}\,. \end{aligned} \tag{3.2}$$

Bulk-to-bulk propagators, which occur in exchange graphs can have any spin occurring in the table of descendents. For simplicity, we concentrate on the scalar bulk-to-bulk propagator $G_\delta(z, w)$ [41]. We shall make use here only of its asymptotic behavior as one of the bulk points approaches the boundary, and we have

$$G_\Delta(z, w) \to \frac{2w_0^\Delta}{(2\Delta - d)(2\Delta - d - 1)} K_\Delta(\vec{w}, z) \qquad \text{as} \qquad w_0 \to 0 \tag{3.3}$$

Generally, supergravity vertices may involve derivative couplings. It was shown in [4] that the action of derivatives on propagators may always be converted into non-derivative couplings up to a multiplicative factor, which may vanish, but which will never diverge.

3.1 Contact graphs

The convergence properties of AdS integrals associated with contact interactions and non-derivative coupling is simple. We consider a correlator for operators of dimensions Δ, Δ_i, $i = 1, \ldots, n \geq 2$, where each of the dimensions obeys the AdS unitarity bound $\Delta \geq d/2$, and we assume that $\Delta \geq \Delta_i$ for all i. We now regularize the AdS integral by keeping Δ_i set at their BPS values, while allowing Δ to be a general complex number in the neighborhood of its BPS value. The contact AdS integral for a correlator with non-coincident points x, x_i, is given by

$$I(\Delta, \Delta_i) = \int \frac{d^{d+1}z}{z_0^{d+1}} K_\Delta(x,z) \prod_{i=1}^{n} K_{\Delta_i}(x_i, z) \,. \tag{3.4}$$

By the unitarity bound on the Δ's, the integral is convergent in the region $z \to \infty$, and in view of (3.2), it is convergent as well when z tends to any boundary point different from x and x_i. As $z \to x_j$, we have

$$I(\Delta, \Delta_i) \sim \frac{C_\Delta C_{\Delta_j}}{(x-x_j)^{2\Delta}} \prod_{i \neq j}^{n} \frac{C_{\Delta_i}}{(x_i - x_j)^{\Delta_i}} \times \int \frac{d^{d+1}z}{z_0^{d+1}} \frac{z_0^{\Delta+\Delta_1+\cdots+\Delta_n}}{(z_0^2 + (z-x_j)^2)^{\Delta_j}} \,. \tag{3.5}$$

This integral is convergent around $z \to x_j$ as long as $\Delta + \Delta_1 + \cdots + \Delta_n - 2\Delta_j > 0$, which is guaranteed here by the fact that $\Delta \geq \Delta_i$ for all $i = 1, \ldots, n$. As $z \to x$ on the other hand, we have

$$I(\Delta, \Delta_i) \sim C_\Delta \prod_{i=1}^{n} \frac{C_{\Delta_i}}{(x_i - x)^{2\Delta_i}} \times \int \frac{d^{d+1}z}{z_0^{d+1}} \frac{z_0^{\Delta+\Delta_1+\cdots+\Delta_n}}{(z_0^2 + (z-x)^2)^{\Delta}} \,. \tag{3.6}$$

This integral is convergent as $z \to x$ as long as $\Delta < \Delta_1 + \cdots + \Delta_n$. Now, the group theory of the R-symmetry group (which coincides with the isometry group of the sphere in AdS$\times$S) guarantees that any correlator with $\Delta > \Delta_1 + \cdots + \Delta_n$ vanishes identically. Thus, there is only one relevant divergence of (3.6) which occurs precisely at $\Delta = \Delta_1 + \cdots + \Delta_n$. The corresponding pole may be extracted exactly, and we have

$$\text{pole } I(\Delta, \Delta_i) = \frac{1}{\Delta_1 + \cdots + \Delta_n - \Delta} \times \prod_{i=1}^{n} \frac{C_{\Delta_i}}{(x-x_i)^{2\Delta_i}} \,. \tag{3.7}$$

as represented on figure 1. Notice that the divergence structure and even the value of the pole divergence (up to the normalizations C_{Δ_i}) are completely independent of the AdS-space dimension d.

3.2 Exchange graphs

The analysis of the convergence structure for exchange diagrams is completely analogous to that of the contact terms. Remarkably, the structure is again independent

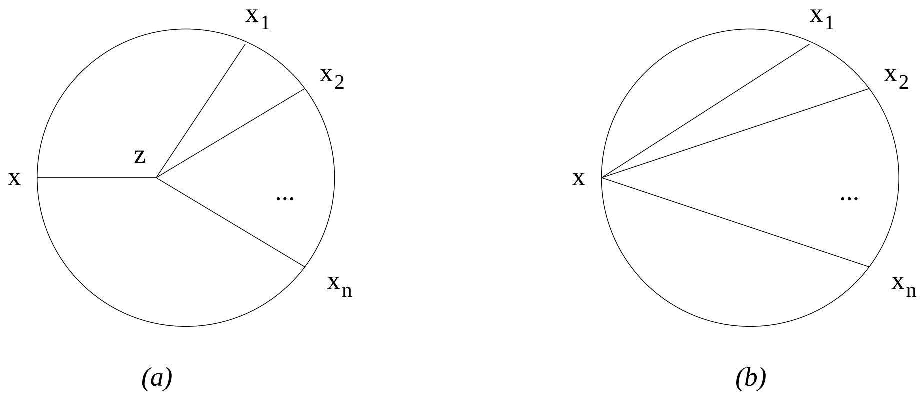


Figure 1: Contact graph (a); Factorized residue at $\Delta = \Delta_1 + \ldots + \Delta_n$ (b).

of the AdS-space dimension d, and we may carry over to general d the criteria of divergence given in [17]. For an exchange graph with boundary points x_i, $i = 1, \ldots, n$ and superconformal primary (scalar) operators of dimensions Δ_i at these points, we thus have the following criteria.

A simple pole divergence occurs in the z-integration over AdS_{d+1} if and only if the following two conditions are satisfied:

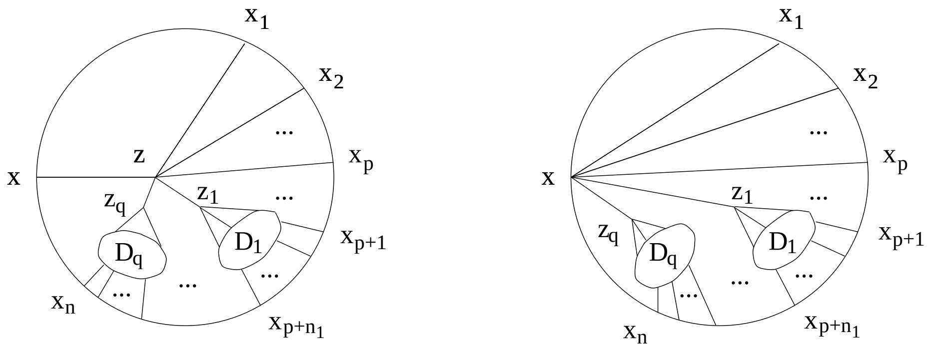
- Either z approaches a point x_i that is connected to z by a boundary-to-bulk propagator or it approaches a point x_i that is connected to z by a string of propagators and bulk interaction points z_a and all points z_a also approach x_i;

- The vertex at z is extremal and the highest dimension of the fields entering the vertex is the one of the field that connects z (directly or through a string of propagators) with x_i.

The residue of the poles may be calculated in a recursive way. We begin with the z-integration over the bulk vertex that is connected to the external operator of the largest dimension Δ. To have a pole, this vertex must be extremal, and Δ must be larger than any of the dimensions of propagators emanating from the z-vertex, including bulk-to-bulk propagators. The corresponding exchange amplitude may then be represented by

$$E(\Delta, \delta_a, \Delta_i) = \int \frac{d^{d+1}z}{z_0^{d+1}} K_\Delta(x, z) \prod_{i=1}^{p} K_{\Delta_i}(x_i, z) \prod_{a=1}^{q} \int \frac{d^{d+1}z_a}{(z_a)_0^{d+1}} G_{\delta_a}(z, z_a) D_a(z_a, \{x_j\}_a)\,, \tag{3.8}$$

where $D_a(z_a, \{x_j\}_a)$ is a reduced amplitude with $n_a + 1$ external legs, graphically represented in figure 2, and $p + \sum_{a=1}^{q} n_a = n$. As the z-vertex is assumed to be extremal, we have $\Delta = \Delta_1 + \cdots + \Delta_p + \delta_1 + \cdots + \delta_q$, and we shall allow Δ to relax away slightly from this value so as to suitably analytically the AdS integrals. The only divergence of the z-integral arises when $z \to x$, which we analyze in parallel to



(a) (b)

Figure 2: Exchange graph (a); Factorized residue at $\Delta = \Delta_1 + \ldots + \Delta_p + \delta_1 + \ldots + \delta_q$ (b).

the case of the contact graph. Using the asymptotics of (3.2) and (3.3), we isolate the only divergence of this integral, which occurs when $\Delta = \Delta_1 + \cdots + \Delta_p + \delta_1 + \cdots + \delta_q$. The pole part is

$$\begin{aligned} \mathrm{pole}E(\Delta, \delta_a, \Delta_i) = & \frac{1}{\Delta - \sum_a \delta_a - \sum_i \Delta_i} \times \prod_{i=1}^{p} \frac{C_{\Delta_i}}{(x_i - x)^{2\Delta_i}} \times \\ & \times \prod_{a=1}^{q} \frac{2}{(2\delta_a - d)(2\delta_a - d - 1)} \int \frac{d^{d+1} z_a}{(z_a)_0^{d+1}} K_{\delta_a}(x, z_a) D_a(z_a, \{x_j\}_a) \, . \end{aligned} \tag{3.9}$$

This result is graphically represented in figure 2. Each factor of the residue may now be treated iteratively using the same formulas. With the help of it, we shall analyze the factorization properties of near-extremal correlators in the next sections.

4. Extremal correlators

Extremal correlators of 1/2 BPS operators $\mathcal{O}_\Delta$ of dimension Δ are of the form

$$\langle \mathcal{O}_\Delta(x) \mathcal{O}_{\Delta_1}(x_1) \cdots \mathcal{O}_{\Delta_n}(x_n) \rangle \qquad \Delta = \Delta_1 + \cdots + \Delta_n \, . \tag{4.1}$$

We shall present arguments below that extremal correlation functions factorize into a product of 2-point functions as follows

$$\langle \mathcal{O}_\Delta(x) \mathcal{O}_{\Delta_1}(x_1) \cdots \mathcal{O}_{\Delta_n}(x_n) \rangle = A(\Delta, \Delta_i; N) \prod_{i=1}^{n} \frac{1}{(x - x_i)^{2\Delta_i}} \, , \tag{4.2}$$

where the overall correlator strength A may be expressed solely in terms of 2-point functions of 1/2 BPS operators. The first part of the argument is based on the AdS structure, and leads to the factorized form of the correlator, while the second is based on the OPE and allows us to relate the overall strength of the correlator to 2- and 3-point functions of 1/2 primary operators. From the AdS arguments, we also obtain the conjectures on the extremal supergravity couplings, as given in (1.3) of section 1.

4.1 AdS argument

The fundamental assumption in the AdS argument is that M-theory on the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds is a finite theory at tree level, and that a finite Maldacena dual conformal field theory at the boundary of AdS exists. The argument then proceeds inductively on the number $n+1$ of operators entering the correlator (4.1).

For $n = 2$, the extremal three point function must factorize by conformal invariance alone,

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\rangle = A(\Delta, \Delta_1, \Delta_2; N) \prod_{i=1}^{2} \frac{1}{(x-x_i)^{2\Delta_i}} . \tag{4.3}$$

We also know from explicit supergravity calculations on $AdS_4 \times S^7$ in and $AdS_7 \times S^4$ in [38, 39] that the extremal supergravity couplings vanish. The product of the vanishing extremal supergravity coupling with the divergent AdS integral accounts for the finite extremal correlator of (4.3). As shown in [10], one may turn this argument around: the divergence of the extremal AdS integral together with the finiteness assumption of M-theory on $AdS_4 \times S^7$ and $AdS_7 \times S^4$ imply that the extremal supergravity coupling $\mathcal{G}(\Delta_1 + \Delta_2, \Delta_1, \Delta_2) = 0$.

For $n = 3$, the extremal 4-point function, we have two types of AdS graphs: a contact graph and exchange graphs. Purely from SO(8) and USp(4) R-symmetry group theory for the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds, we know that all bulk interaction vertices in each of these graphs have to be extremal themselves. The two extremal 3-point couplings in the exchange graphs produce two zeros which are multiplied by doubly divergent AdS integrals. The result is finite and the pole contributions from each AdS integral localize each bulk vertex at the point x on the boundary. The result from the exchange graphs is a finite contribution of the factorized form (4.2). There remains an extremal contact graph, whose AdS integral diverges. Assuming finiteness again, this implies that the extremal supergravity 4-point coupling must vanish, as indeed conjectured in (1.3).

Next, assuming that all extremal p-point supergravity couplings vanish for $p \le n$, it is easy to show that they must vanish for all $p \le n+1$. All bulk interaction vertices for exchange graphs are at most n-point couplings and must be themselves extremal, and thus vanish by the assumption of induction. Picking up the poles of the divergent AdS integrals, one finds again a finite factorized form as in (4.2). The remaining contact graph involves an $n+1$-point extremal supergravity coupling, which is not yet known to vanish. However, the divergence of the AdS integral together with the finiteness assumption again forces also this extremal supergravity coupling to vanish, proving the induction statement at order $n+1$.

Thus, there only remain the exchange graphs, in which every bulk vertex is now extremal. Starting with the highest dimension boundary-to-bulk propagator, we apply the recursive pole expressions of (3.9) and cancel the vanishing extremal

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coupling by the pole of the AdS integral. The corresponding bulk vertex collapses onto the boundary and the process can now be repeated for each of the factors. There results, at the end of this recursive process the completely factorized form of (4.1), with an overall factor A which is independent of x_i.

4.2 Operator product expansion argument

In this subsection we show that, assuming the factorized space-time form of the extremal correlators as in (4.2), the overall normalization $A(\Delta, \Delta_i; N)$ is related to 2- and 3-point functions of 1/2 BPS operators.

Recall that the "single color trace" scalar operators $\mathcal{O}_k(x)$ have dimension $\Delta = \mathcal{K}k$, where $\mathcal{K} = 1/2, 1, 2$ for $AdS_4 \times S^7$, $AdS_5 \times S^5$ and $AdS_7 \times S^4$ respectively, while their R-symmetry Dynkin labels are $(k, 0, 0, 0)$ of SO(8) for $AdS_4 \times S^7$, $(0, k, 0)$ of SU(4) for $AdS_5 \times S^5$ and $(0, k)$ of USp(4), for $AdS_7 \times S^4$. They couple directly to the supergravity and Kaluza-Klein modes with the corresponding quantum numbers. By taking the OPE of such operators with suitable projections applied, we obtain further "multiple color trace" scalar operators with the same quantum numbers. There exists an independent operator for every partition of Δ into a sum of dimensions Δ_i of single color trace operators obeying the unitarity bound $\Delta_i \geq d/2$. We shall denote such partitions $\pi(\Delta)$ or simply by π when no confusion is possible. The corresponding operators are defined by

$$\begin{aligned} &\mathcal{O}_{\pi(\Delta)}(x) = [\mathcal{O}_{\Delta_1}(x) \cdots \mathcal{O}_{\Delta_p}(x)]|_{\text{proj } 1/2 \ BPS} \\ &\pi(\Delta) : \Delta = \Delta_1 + \cdots + \Delta_p \,. \end{aligned} \tag{4.4}$$

It is customary to normalize the single trace operators $\mathcal{O}_\Delta(x)$ by their 2-point functions

$$\langle \mathcal{O}_\Delta(x) \mathcal{O}_{\Delta'}(y) \rangle = \frac{\delta_{\Delta,\Delta'}}{(x-y)^{2\Delta}} \,. \tag{4.5}$$

The normalization of the operators $\mathcal{O}_{\pi(\Delta)}$ is then determined by the dynamics of the theory and given by

$$\langle \mathcal{O}_{\pi(\Delta)(x)} \mathcal{O}_{\sigma(\Delta')}(y) \rangle = \frac{\delta_{\Delta,\Delta'} M_{\pi,\sigma}}{(x-y)^{2\Delta}} \,. \tag{4.6}$$

For given Δ, the matrix M of all partitions is positive definite, symmetric and depends only upon Δ and the number of colors N.

First, assuming the factorized form for the extremal correlators of operators $\mathcal{O}_\Delta$, as in (4.2), the coefficient A is given by

$$A(\Delta, \Delta_i; N) = M_{\pi,\sigma} \,, \qquad \pi(\Delta) = \Delta \,, \qquad \sigma(\Delta) = \Delta_1 + \cdots + \Delta_n \,. \tag{4.7}$$

This follows directly by letting all points $x_i \to y$ and then using the definition of $\mathcal{O}_{\pi(\Delta)}$ and the normalization of these operators.

Second, the argument may be generalized to the case of a correlation function of multi trace operators, characterized by non-trivial partitions $\pi(\Delta)$. We assume that the factorized form holds for single trace operators $\mathcal{O}_\Delta$. This is enough to show that it holds also for multi-trace operators. Suppose we wish to evaluate the correlator

$$\langle \mathcal{O}_{\pi(\Delta)}(x)\mathcal{O}_{\pi_1(\Delta_1)}(x_1)\cdots\mathcal{O}_{\pi_n(\Delta_n)}(x_n)\rangle\,, \qquad \Delta=\Delta_1+\cdots+\Delta_n\,, \tag{4.8}$$

for given partitions

$$\pi(\Delta)=\delta^{(1)}+\cdots+\delta^{(p)}\,, \qquad \pi_i(\Delta_i)=\delta_i^{(1)}+\cdots+\delta_i^{(p_i)}\,. \tag{4.9}$$

Clearly, this correlation function can be obtained from the correlator involving $\mathcal{O}_{\pi(\Delta)}$ and products of single trace operators only,

$$\left\langle \mathcal{O}_{\pi(\Delta)}(x)\prod_{i=1}^{n}\prod_{a=1}^{p_i}\mathcal{O}_{\delta_i^{(a)}}(x_{i,a})\right\rangle \tag{4.10}$$

by letting $x_{i,a}\to x_i$ for all $a=1,\ldots,p_i$, with unit coefficient of proportionality between the two correlators.

Third, it remains to link the operator of maximal dimension $\mathcal{O}_{\pi(\Delta)}$ to single trace operators, in such a way that the correlator may be evaluated from extremal correlators of single trace operators alone. One may be tempted to view $\mathcal{O}_{\pi(\Delta)}(x)$ as the composite of $\mathcal{O}_{\delta^{(1)}}(x)\cdots\mathcal{O}_{\delta^{(1)}}(x)$, but this would give rise to a correlation function which is not, in general, extremal. Instead, one must reconstruct the operator $\mathcal{O}_{\pi(\Delta)}$ from the most singular OPE term in the operator product expansion of operators of a string of single trace operators, as follows

$$\mathcal{O}_{\Delta+\delta}(x)\mathcal{O}_{\tau(\delta)}(y)=\frac{1}{(x-y)^{2\delta}}\sum_{\pi}\Lambda_{\tau,\pi}\mathcal{O}_{\pi(\Delta)}(x)+\text{less singular terms}\,, \tag{4.11}$$

where $\Lambda_{\tau,\pi}$ is a matrix that depends upon Δ, δ and N. It may be calculated from the extremal 3-point function and the matrix M of 2-point functions. The dimension δ is taken sufficiently large so that the relation may be inverted.

5. General near-extremal correlators

We now discuss the case of near-extremal correlators of the form

$$\langle \mathcal{O}_{\Delta}(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle\,, \qquad \Delta+2m\mathcal{K}=\Delta_1+\cdots+\Delta_n\,, \tag{5.1}$$

where m is an integer with $0\le m\le n-2$, and $\mathcal{K}$ is the unit of dimension for superconformal primaries, namely $\mathcal{K}=1/2$ for $AdS_4\times S^7$, $\mathcal{K}=1$ for $AdS_5\times S^5$ and $\mathcal{K}=2$ for $AdS_7\times S^4$. The integrality of m follows from the conservation of the $\mathbb{Z}_2$ subgroup in the center of the R-symmetry groups SO(8), SO(6) or USp(4). We shall generically denote such correlators by E^m_{n+1}. For $m = 0$, we recover the extremal correlation functions, which were treated already in the preceding section. Next-to-extremal correlators have $m = 1$, and the bound above starts at 4-point correlators. Next-to-next-to-extremal correlators necessitate 5 points or more, and so on.

In the $AdS_5 \times S^5$ case, it was conjectured that such correlators E^m_{n+1} decompose into a sum of products of non-renormalized 2- and 3-point functions, and (for $m \geq 2$) renormalized higher-point functions. This decomposition property has been checked on the gauge theory side up to order g^2_{YM} in [17]. Precisely the same structure emerges from the contribution of the exchange diagrams (for $n \geq 4$) on the AdS side. The contact graph, which also arises on the AdS side, would spoil this decomposition property, as it cannot be written as a sum of factorized contributions in a non-trivial way. Assuming that the decomposition property found at weak coupling extends to strong coupling, we are naturally led to conjecture that the supergravity couplings $\mathcal{G}(\Delta, \Delta_1, \ldots, \Delta_n)$ should vanish. This has been shown by direct supergravity calculation for next-to-extremal 4-point couplings in [12].

Clearly, the arguments in favor of the vanishing of the sub-extremal supergravity couplings ($m \geq 1$) are not quite as compelling as the arguments given in favor of the vanishing of the extremal correlators, where finiteness criteria played a deciding role. Nonetheless, vanishing in the sub-extremal case appears to hold true and produces a compelling global picture for structure of near-extremal correlators. In the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ cases of interest to us, we do not have a *weak* coupling description at our disposal. What we do have is the fully interacting strong coupling superconformal $\mathcal{N} = 8$ and $(0, 2)$ field theories on the one side and the zero couplling (free) theories on the other. But there is no family of superconformal (or even conformal) field theories continuously connecting the two. By analogy with the $\mathcal{N} = 4$ case of [17], it is clear that the free $\mathcal{N} = 8$ or $(0, 2)$ theories yield the proposed decomposition of the correlators. Since the free theory is disconnected from the fully interacting theory of interest though, we cannot draw much support for the proposed decomposition of the correlators from it.

The situation is however more promising on the AdS side: in complete analogy with [17], we can show that the exchange diagrams decompose into a sum of products of lower order correlation functions, assuming that near-extremal supergravity couplings vanish. The decomposition may be schematically represented as

$$E^m_n|_{\text{exchange}} = \sum_{\{n_j, m_j\}} \prod_{i=1}^{n-m-1} E^{m_i}_{n_i}\,, \qquad \text{with} \qquad \sum_{i=1}^{n-m-1} n_i = 2(n-1) - m\,, \tag{5.2}$$

with $\sum_{i=1}^{n-m-1} = m$. The restriction $m \leq n - 3$ ensures that each exchange diagram decomposes into a sum of terms, each of which has at least two factors, so that $n_i < n - 1$. The arguments are completely parallel to those given in [17], so we shall limit ourselves here to presenting the cases of next-to-extremal correlators E^1_{n+1}.

We consider the next-to-extremal $n+1$-point functions

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle\,, \qquad \text{with} \qquad \Delta = \Delta_1+\cdots+\Delta_n+2\mathcal{K}\,. \tag{5.3}$$

We begin with $n=3$ and assume that the states propagating in one of the exchange diagrams is a superconformal primary with dimension Δ_e. Group theory requires

$$\Delta \le \Delta_e + \Delta_1 \le (\Delta_2+\Delta_3)+\Delta_1 = \Delta + 2m\mathcal{K}\,, \tag{5.4}$$

with equality for extremal couplings. One of the two vertices has to be extremal and hence have vanishing coupling. If the vertex $(\Delta\Delta_1\Delta_e)$ is extremal, the vanishing coupling cancels the pole as the vertex approaches the boundary, leaving a factorized result $\langle\mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_1}\rangle\langle\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_2+\Delta_3+2\mathcal{K}}\rangle$. If it is the other vertex, the integral is finite and the net result is zero. We thus have $E^1_4 = E^0_2 E^1_3$. Exchange of descendants may be treated as in [17], and do not modify the picture. The case E^1_n is very similar: the only contribution comes when the vertex linking the operator of highest dimension $\mathcal{O}_\Delta$ to the tree is extremal and approaches the boundary. The diagram then splits into two factors, one extremal and the other next-to-extremal. The latter further decomposes until one is left with one 3 point function and $n-2$ two-point functions, the space-time dependence of both of these being fixed by conformal symmetry alone.

The AdS integral for the extremal contact graph is divergent and finiteness of tree-level supergravity thus forces the extremal supergravity coupling to vanish. For next-to-extremal correlators, the contact graph is finite, and merely spoils the factorizability of the correlator. In analogy with the $AdS_5\times S^5$ case, it is likely that the factorization of near-extremal correlators is a consequence of supersymmetry, and hence that the contact term should vanish on those grounds. On the AdS side, it is remarkable that all exchange diagrams exhibit the factorizability property, while the single contact graph would not.

As was shown in [17], the same reasoning applies to general near-extremal correlators. Again, their exchange graphs factorize provided the contact graphs at lower orders are absent. We are thus led to conjecture the vanishing of near-extremal supergravity couplings

$$\mathcal{G}(\Delta,\Delta_1,\ldots,\Delta_n)=0\,, \qquad \Delta+2m\mathcal{K} = \Delta_1+\cdots+\Delta_n\,, \tag{5.5}$$

for $m \le n-2$. It would be interesting to verify that statement at the level of 4-point functions by adapting the analysis of [12] to the $AdS_4\times S^7$ or $AdS_7\times S^4$ case.

Assuming the above conjecture, the space-time form of the next-to-extremal correlators may be written down exactly, up to a number of space-time independent couplings $A^{(n)}_{ij}(\Delta,\Delta_1,\ldots,\Delta_n;N)$

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_{\Delta_1}(x_1)\cdots\mathcal{O}_{\Delta_n}(x_n)\rangle = \sum_{i<j}^n A^{(n)}_{ij}\frac{(x-x_i)^{2\mathcal{K}}(x-x_j)^{2\mathcal{K}}}{(x_i-x_j)^{2\mathcal{K}}}\prod_{k=1}^n\frac{1}{(x-x_k)^{2\Delta_k}}\,. \tag{5.6}$$

Using the OPE of two of the operators, one may relate the couplings $A_{ij}^{(n)}(\Delta, \Delta_1, \ldots, \Delta_n; N)$ to the 2- and 3-point couplings of single- and multi-trace 1/2 BPS operators.

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